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SELECTION OF PARAMETERS FOR A DYNAMIC SYSTEM
UNIVERSAL FOR A GIVEN GROUP OF MANEUVERS

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universal'noy dlya zadannogo klassa manevrov"

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ABSTRACT

In the common formulation of optimum control it is necessary to derive such control functions and select such values of control which would permit the transfer of the dynamic system from a given initial condition into a given final condition for a fixed period of time under the maximum (minimum) of some functional. /1 *

Optimum control and optimum values of the parameters which are the results of the formulated problem's solution will depend in the general case upon maneuver parameters (initial and final values, time of maneuver completion). The optimum accomplishment of a large number of maneuvers with different parameters will require creation of a large number of nonidentical systems.

The problem is stated of using a given number of various maneuvers by means of a given number of system types less than the number of maneuvers. The value of the initial functional, averaged on the base of all maneuvers, is taken as a criterion of optimization. Distribution of the required number of accomplished maneuvers according to their parameters is taken to be known. /2

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* Numbers given in margin indicate pagination in original foreign text.

An alternative statement of the problem of universalization of optimum parameters for the dynamic system is considered. It is assumed that the system may consist of separate modules. The problem is stated of the selection of optimum parameters for a module, universal for all given maneuvers, and of optimum number of modules for each maneuver.

Examples are considered for solution in application to the problem of maximum payload delivery when a body of variable mass and restricted jet power moves in a gravitational field. It is shown that many maneuvers can be accomplished with a little decrease of the averaged payload by means of one type of propulsion system. Two types of propulsion system permit us practically to eliminate the decrease. When a universal module is used, the sacrifice is equally small for different changes of maneuver parameters.

1. Let us assume that the behavior of a dynamic system is described by the following ordinary equations

$$\dot{x}_i = f_i(x_j, u_k, w_l) \quad (i, j = 1, \ldots, n;\ k = 1, \ldots, r;\ l = 1, \ldots, q) \tag{1.1}$$

where x_i represents the phase coordinates of the system, $u_k(t)$ the control functions, w_l the constant control parameters, and the point denotes time differentiation t. /3

The usual formulation of the optimum control problem requires the construction of such $u_k(t)$ controls and the selection of such w_l parameters from the admissible group that would facilitate that transfer of system (1.1) from a given initial condition

$$x_i(0) = x_{i0} \qquad (i = 1, \dots, n) \tag{1.2}$$

into a given final condition

$$x_i(T) = x_{i1} \qquad (i = 1, \dots, n) \tag{1.3}$$

for a fixed period of time T under a maximum (minimum) functional

$$x_{01} = x_0(T) \qquad \left(\dot{x}_0 = f_0(x_j, u_k, w_l),\ x_0(0) = 0 \right) \tag{1.4}$$

The optimum $u_k(t)$ controls and optimum values of the w_l parameters, re- 14
presenting a solution of the formulated problem will generally depend on the nature of the manuever $\{T, x_{i0}, x_{i1}\ (i = 1, \dots, n)\}$. We will assume that dynamic systems (1.1) are different if the corresponding parameters of w_l do not coincide. It is assumed that various $U_k(t)$ controls may occur in the same system (with the same limitations, of course).

The optimum performance (in a sense of the (1.4) functional) of a large number of maneuvers with various characteristics

$$\{T^{(s)}, x_{i0}^{(s)}, x_{i1}^{(s)}\ (i = 1, \dots, n)\} \qquad (s = 1, \dots, S) \tag{1.5}$$

requires the creation of a large number of nonidentical systems. The latter may prove to be unprofitable from an economic point of view.

The problem, as stated, is to achieve S various maneuvers by means of a given number of Ω of system types (1.1) less than S $(1 \leq \Omega < S)$. The values of functional (1.4), averaged on the basis of all maneuvers, is used as a criterion of optimization.

The optimum Ω number of the system types should be determined from the minimum cost. This calls for a knowledge of the development cost of system (1.1) with parameters different from the previous ones, and a correspondence between the cost and functional (1.4). The latter problem is not discussed in this study--the Ω number of system types is assumed to be preassigned. /5

There is an alternative formulation of the problem regarding the universaliaation of optimum system parameters. If system (1.1) can be made up of modules, its w_l parameters may be expressed as follows

$$w_l = w_l(\sigma_l, \Delta w_l) \qquad (\sigma_l = 1, 2, 3, \ldots;\ l = 1, \ldots, q) \tag{1.6}$$

where ΔW_l represents the module parameters (in the particular case of fully autonomous modules, dependence (1.6) will be linear: $w_l = \sigma_l \Delta w_l$). The problem is to select the optimum Δw_l parameters of the modules, common to all S types of maneuvers, and the optimum number of σ_l modules for each maneuver.

2. The probability approach is used in the formulation of the optimization criterion for the two above-stated problems. The distribution of the required number of maneuvers to be carried out according to their parameters (1.5) is predefined. If the required number of completed maneuvers of each type is added to the total number of all types of maneuvers, this distribution could be treated as a distribution of the maneuver probability by the following type

$$p_s = p_s(T^{(s)}, x_{i0}^{(s)}, x_{i1}^{(s)}) \qquad \left(s = 1, \ldots, S;\ \sum_{s=1}^{S} p_s = 1,\ i = 1, \ldots, n\right) \tag{2.1}$$

The mean value of functional (1.4) will then figure as a criterion of /6
optimization

$$\langle x_{01} \rangle = \sum_{s=1}^{S} p_s x_0^{(s)} (T^{(s)}) \tag{2.2}$$

that is the value of functional (1.4) averaged up on the basis of all types of maneuvers (1.5).

The maneuver parameters may be preset not discretely, as in (1.5), but continuously

$$T = T(s), \; x_{i0} = x_{i0}(s), \; x_{i1} = x_{i1}(s) \quad (i = 1, \ldots, n; \; 0 \leq s \leq S) \tag{2.3}$$

where S may assume all the values from interval (o, S) (not only integers). In this case, the probability distribution (2.1) is replaced by the density of the probability distribution

$$\rho = \rho(s) \qquad \left(0 \leq s \leq S, \; \int_0^S \rho(s) ds = 1 \right) \tag{2.4}$$

and functional (2.2) is changed to

$$\langle x_{01} \rangle = \int_0^S \rho(s) x_0 (T(s), s) ds \tag{2.5}$$

3. Let W_l parameters of system (1.1) be capable of assuming Ω values

$$w_l^{(\omega)} \qquad (\omega = 1, \ldots, \Omega)$$

It is required to perform S types of maneuvers(1.5) with a (2.1) probability distribution by insuring a maximum (minimum) (2.2) functional. /7

We will introduce the following new independent variable for each S-type maneuver in place of time t

$$\tau = t / T^{(s)} \tag{3.1}$$

in such a way as to reduce the finite conditions (1.3) and functional (2.2) to point $\tau = 1$ which is common to all maneuvers.

Thereafter the problem can be reduced to a standard formulation (1.1)-(1.4) for an expanded system of differential equations:

$$\frac{dx_i^{(s)}}{d\tau} = T^{(s)} f_i\left(x_j^{(s)}, u_k^{(s)}, w_l^{(\omega)}\right) \quad \left(\begin{array}{c} 0 \leq \tau \leq 1;\ s=1,\dots,S \\ \omega=1,\dots,\Omega<S;\ l=1,\dots,q \\ i=0,1,\dots,n;\ j=1,\dots,n;\ k=1,\dots,r \end{array}\right) \tag{3.2}$$

with the following boundary conditions

$$x_0^{(s)}(0)=0,\ x_i^{(s)}(0)=x_{i0}^{(s)},\ x_{i1}^{(s)}(1)=x_{i1}^{(s)} \qquad (s=1,\dots,S;\ i=1,\dots,n) \tag{3.3}$$

and functional

$$\langle x_{01}\rangle = \sum_{s=1}^{S} p_s x_0^{(s)}(1) \tag{3.4}$$

Here, as before, $x_i^{(s)}$ represents the phase coordinates, (S (n + 1) units), $U_k^{(s)}(\tau)$ are the control functions, and /S z units/, $w_l^{(w)}$ the constant /8

control parameters (Ω_q units).

In the case of the constant distribution (2.3)-(2.5), we arrive at the two-dimensional variational problem for the following system

$$\frac{\partial x_i}{\partial \tau} = T(s) f_i(x_j, u_k, w_l) \quad \begin{pmatrix} 0 \leq \tau \leq 1 \; ; \; 0 \leq s \leq S \\ i = 0, 1, \ldots, n \; ; \; j = 1, \ldots, n \\ k = 1, \ldots, z \; ; \; l = 1, \ldots, q \end{pmatrix} \tag{3.5}$$

with the following finite conditions

$$x_0(0,s) = 0, \; x_i(0,s) = x_{i0}(s), \; x_i(1,s) = x_{i1}(s) \qquad (i = 1, \ldots, n) \tag{3.6}$$

and functional

$$\langle x_{01} \rangle = \int_0^S \rho(s)\, x_0(1,s)\, ds \tag{3.7}$$

Here $x_i = x_i(\tau, s)$, $u_k = u_k(\tau, s)$ are phase coordinates and control functions depending on two variables; $W_l = W_l(s)$ are piecewise constant and control functions with a preset number of levels $W_l^{(w)}$ ($w = 1, \ldots, \Omega$) which depend only on s.

4. The above-cited variational formulations of (3.2)-(3.4) and (3.5)- /9 (3.7)are applied to the universal module problem with some changes. Boundary conditions (3.3) and (3.6) and functionals (3.4) and (3.7) remain unchanged.

There is some change in the writing of the differential equations (for the sake of simplicity of recording, the (1.6) dependence is assumed to be linear):

for a discrete distribution of (3.2)

$$\frac{dx_i^{(s)}}{d\tau} = T^{(s)} f_i(x_j^{(s)}, u_k^{(s)}, \sigma_l^{(s)} \Delta W_l) \quad \begin{pmatrix} 0 \leqslant \tau \leqslant 1;\ s = 1,\dots,S;\ i = 0,1,\dots,n \\ j = 1,\dots,n;\ k = 1,\dots,r;\ l = 1,\dots,q \end{pmatrix} \tag{4.1}$$

for a continuous distribution of (3.5)

$$\frac{\partial x_i}{\partial \tau} = T(s) f_i(x_j, u_k, \sigma_l \Delta W_l) \quad \begin{pmatrix} 0 \leqslant \tau \leqslant 1;\ 0 \leqslant s \leqslant S;\ i = 0,1,\dots,n \\ j = 1,\dots,n;\ k = 1,\dots,r;\ l = 1,\dots,q \end{pmatrix} \tag{4.2}$$

In the (4.1) equations the constant control parameters ΔW_l are the same for all s; $\sigma_l^{(s)} = 1,2,3,\dots$ integers may be different for each s number (unlike (3.2) where the Ω number of parameters $W_l^{(w)}$ is less than the S number of the equation groups).

In the (4.2) equations ΔW_l is the constant control parameter $(\partial \Delta W_l/\partial \tau = \partial \Delta W_l/\partial s = 0)$; $\sigma_l = \sigma_l(s)$ the piecewise constant control function of one s variable which can assume any integral values (unlike (3.5) where the number of Ω levels of the piecewise-constant control $W_l(s)$ is predefined). /10

5. Let us consider the problem of delivering a maximum payload by a variable-mass body moving in a gravitational field with limited jet power. Assuming an ideally controlled engine, equations (1.1) are recorded as follows (see review (1))

$$\dot{G}_\sigma = -\frac{(G_\sigma + G_\nu)^2}{G_\nu}\,\frac{\alpha}{2g}\,\frac{a^2}{N}$$
$$\dot{\vec{r}} = \vec{v}, \quad \dot{\vec{v}} = a\vec{e} + \vec{R} \tag{5.1}$$

Here $G_\sigma, \vec{r}, \vec{v}$ are phase coordinates: G_σ is the total weight of the current reserve of working medium $G_\mu = G_\mu(t)$ and payload G_π = const., and $\vec{r}$ and $\vec{v}$ the radius vector and velocity. Control functions: $0 \leq a(t) < \infty$ is the acceleration by jet thrust, $0 \leq N(t) \leq 1$ the source power applied to the maximum, and $|\vec{e}(t)| = 1$ the vector unit of the thrust direction. The control parameter $G_\nu = \alpha N_0$ is the weight of the power source ($\alpha = \alpha(G_\nu)$ is the specific gravity of the power source which generally depends on G_ν, and N_0 is the maximum power of the source). The gravitational acceleration at point $(\vec{r}, t)$ and on the Earth's surface is indicated by $\vec{R} = \vec{R}(\vec{r}, t)$ and g.

Initial conditions /11

$$G_\sigma(0) = G_0 - G_\nu, \quad \vec{r}(0) = \vec{r}_0, \quad \vec{v}(0) = \vec{v}_0 \tag{5.2}$$

Finite conditions

$$\vec{r}(T) = \vec{r}_1, \quad \vec{v}(T) = \vec{v}_1 \tag{5.3}$$

Functional of the problem

$$G_\pi = G_\sigma(T) \rightarrow max \tag{5.4}$$

It is a known fact that in this problem the power of the jet stream must always be at a maximum N(t) = 1. Nor does the program of the jet acceleration depend on the engine parameteres: $\vec{a}(t) = a\vec{e}$ function should be selected from the following minimum integral

$$J = \int_0^T a^2 dt \tag{5.5}$$

and should facilitate the transposition of $\left(\ddot{\vec{r}} = \vec{a} + \vec{R}\right)$ between the two preset points $\{\vec{r}_0, \vec{v}_0\}$ and $\{\vec{r}_1, \vec{v}_1\}$ during time T.

The first equation of (5.1) may be integrated in quadratures and resolved in relation to the functional of problem (1)

$$G_T = G_v\left(\frac{1}{G_v + (\alpha/2g)\,J} - 1\right) \tag{5.6}$$

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(here G_π and G_ν are assumed to refer to the initial weight of G_0).

After that, the functional of (5.5) $J = J(T, \vec{r}_0, \vec{v}_0, \vec{r}_1, \vec{v}_1)$ may be considered instead of $\{T, \vec{r}_0, \vec{v}_0, \vec{r}_1, \vec{v}_1\}$ as a characteristic of maneuver (1.5), and the distribution of the maneuvers by type may be considered as predefined in the following form

for a discrete distribution of (2.1)

$$p_s = p_s(J^{(s)}) \qquad \left(s = 1,\dots,S;\ \sum_{s=1}^{S} p_s = 1\right) \tag{5.7}$$

for a continuous distribution of (2.4)

$$\rho = \rho(J) \qquad \left(J_0 \leq J \leq J_1,\ \int_{J_0}^{J_1} \rho(J)\,dJ = 1\right) \tag{5.8}$$

The functional of the problem is written down accordingly:

for a discrete distribution (compare (2.2))

$$\langle G_T\rangle = \sum_{s=1}^{S} p_s G_v\left(\frac{1}{G_v + (\alpha/2g)\,J} - 1\right) \tag{5.9}$$

for a continuous distribution (compare (2.5))

$$\langle G_T \rangle = \int_{\zeta_0}^{\zeta_1} \rho(\zeta)\, G_\nu \left(\frac{1}{G_\nu + (\alpha/2g)\zeta} - 1 \right) d\zeta \qquad (5.10)$$

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If the number of engine types is not restricted, the optimum weight of G_ν will, as is known (1), amount to (with α = const.)

$$G_\nu(\zeta) = \sqrt{\frac{\alpha}{2g}\zeta} - \frac{\alpha}{2g}\zeta$$

The performance of a given group of maneuvers (5.7) and (5.8) by means of a preset $\Omega < S$ number of engine types (see point 3) is reduced to finding the maximum sum of (5.9) with

$$G_\nu = G_\nu^{(\omega)} \qquad (\omega = 1, \ldots, \Omega)$$

or the maximum integral of (5.10) with

$$G_\nu(\zeta) = G_\nu^{(\omega)} \qquad (\omega = 1, \ldots, \Omega)$$

(where Ω is predefined, and optimum $G_\nu^{(w)}$ levels should be selected).

In the case of the problem involving the selection of the optimum parameters of the universal module (see point 4), $\sigma \Delta G_\nu$ should be substituted for G_ν in (5.9) and (5.10), and the specific gravity of the power source α should be assumed as depending on ΔG_ν rather than on G_ν. The functional of (5.9) will then be expressed as

$$\langle G_T \rangle = \Delta G_\nu \sum_{s=1}^{S} p_s \sigma^{(s)} \left(\frac{1}{\sigma^{(s)} \Delta G_\nu + [\alpha(\Delta G_\nu)/2g]\zeta} - 1 \right) \tag{5.11}$$

and the functional of (5.10) as

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$$\langle G_T \rangle = \Delta G_\nu \int_{\zeta_0}^{\zeta_1} \rho(\zeta)\, \sigma \left(\frac{1}{\sigma \Delta G_\nu + [\alpha(\Delta G_\nu)/2g]\zeta} - 1 \right) d\zeta \tag{5.12}$$

In (5.11), $\sigma^{(s)}$ represents arbitrary positive integers, and in (5.12) σ stands for a piecewise-constant function with an arbitrary number of integral levels.

We will designate

$$\Phi = \frac{\alpha_*}{2g}\zeta \quad (0 \leq \Phi \leq 1), \quad \varkappa = \alpha/\alpha_* \quad (\varkappa \geq 1)$$

(α is the limiting value of the specific gravity of the power source, assuming an infinite power) and consider some cases when the maneuvers are distributed continuously within a range of (Φ_0, Φ_1) with equal probability, that is

$$\rho = 1/(\Phi_1 - \Phi_0) \ \text{при}\ \Phi_0 \leq \Phi \leq \Phi_1, \quad \rho = 0 \ \text{when}\ \Phi < \Phi_0, \ \Phi > \Phi_1$$

The dependence of the specific gravity of the power source on its absolute parameters (G_ν, $\partial\alpha/\partial G_\nu < 0$ - см.[1]) does not substantially affect the methods of solution. The introduction of this dependence in a problem involving a fixed number of $G_\nu^{(w)}$ levels does not produce any qualitative changes. In the case of a universal module, however, the $\alpha(\Delta G_\nu)$ or $\chi(\Delta G_\nu)$ dependence is important from a qualitative point of view. With $\chi = 1$ the optimum size of the module coverages to zero, but if $\partial\chi/\partial\Delta G_\nu < 0$, the optimum size of the module is finite. /15

The (5.10) functional of the first problem is therefore recorded as $\chi = 1$.

$$\langle G_T \rangle = \frac{1}{\Phi_1 - \Phi_0} \int_{\Phi_0}^{\Phi_1} G_\nu \left(\frac{1}{G_\nu + \Phi} - 1 \right) d\Phi \tag{5.13}$$

and the (5.12) functional of the second problem as

$$\langle G_T \rangle = \frac{\Delta G_\nu}{\Phi_1 - \Phi_0} \int_{\Phi_0}^{\Phi_1} \sigma \left(\frac{1}{\sigma\Delta G_\nu + \chi(\Delta G_\nu)\Phi} - 1 \right) d\Phi \tag{5.14}$$

The (5.13) integral is used by segment ($\Phi_0^{(w)}$, $\Phi_1^{(w)}$), where the $G_\nu^{(w)}$ level is optimal

$$\langle G_T \rangle = \frac{1}{\Phi_1 - \Phi_0} \sum_{\omega=1}^{\Omega} G_\nu^{(\omega)} \left[\ln \frac{\Phi_1^{(\omega)} + G_\nu^{(\omega)}}{\Phi_0^{(\omega)} + G_\nu^{(\omega)}} - \left(\Phi_1^{(\omega)} - \Phi_0^{(\omega)} \right) \right]$$

The moments of changing from one level to another are determined from the maximum subintegral expression of (5.13). After that the problem is reduced to /16 finding the maximum Ω function of the following variables $\langle G_\pi \rangle = \varphi(G_\nu^{(1)}, \ldots, G_\nu^{(\Omega)})$ provided that the subintegral expression of (5.13) is nonnegative, which is equivalent to the following condition

$$\min_{\omega = 1, \ldots, \Omega} G_\nu^{(\omega)} \leqslant 1 - \Phi_1$$

This procedure can be carried out numerically by the steepest descent method. With $\Omega = 1$, the optimal value of the only $G_\nu^{(i)}$ is defined by the solution of the following equation

$$\frac{\partial \langle G_\pi \rangle}{\partial G_\nu^{(1)}} = \frac{1}{\Phi_1 - \Phi_0} \ln \frac{\Phi_1 + G_\nu^{(1)}}{\Phi_0 + G_\nu^{(1)}} - \frac{G_\nu^{(1)}}{(\Phi_1 + G_\nu^{(1)})(\Phi_0 + G_\nu^{(1)})} - 1 = 0$$

if the root of that equation is found to be the lesser $(1 - \Phi_1)$, otherwise $G_\nu^{(i)} = 1 - \Phi_1$.

Shown in figure 1 is a graph of an averaged payload $< G_\pi >$ for various intervals $\Delta\Phi = \Phi_1 - \Phi_0 = 0;\ 0.2;\ 0.4;\ 0.8$ depending on the initial interval point Φ_0. The dotted curves correspond to the continuous optimum law of change $G_\nu(\Phi) = \sqrt{\Phi} - \Phi$ (for infinite number of engine types see dotted line in figure 2). It appears that one type of propulsion system (curves $\Omega = 1$) can be used to approach very closely to the maximum possible values of a payload. A /17 change to two types of propulsion system would reduce the loss of payload almost

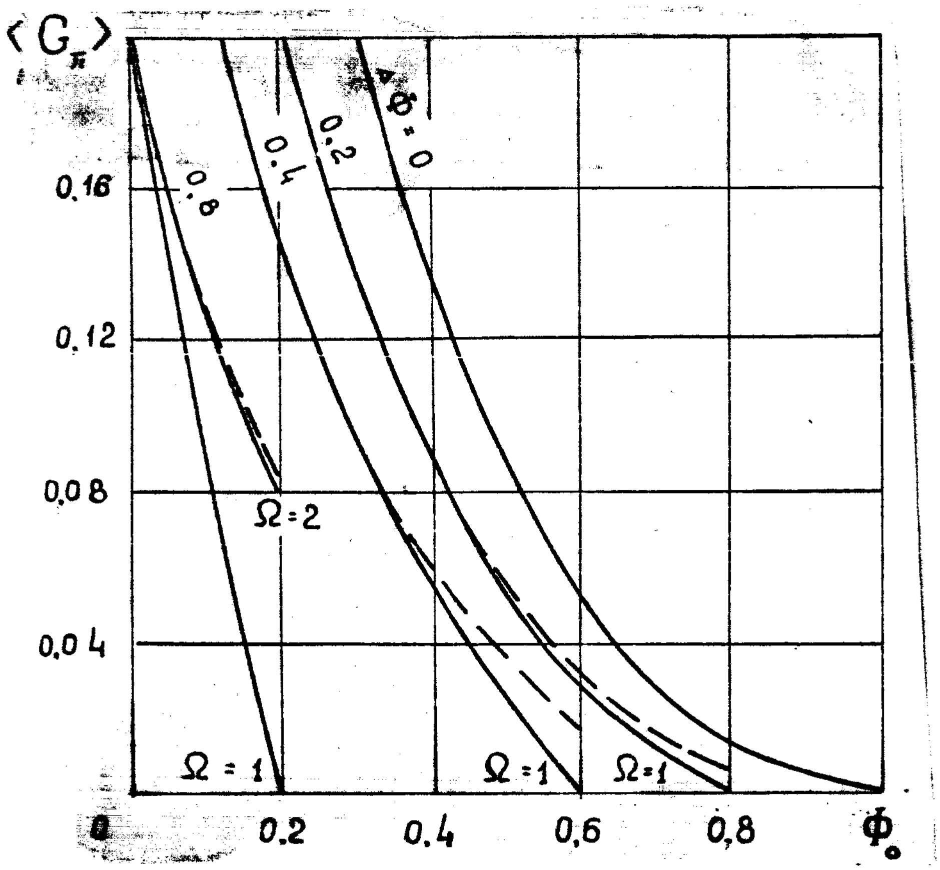

Figure 1

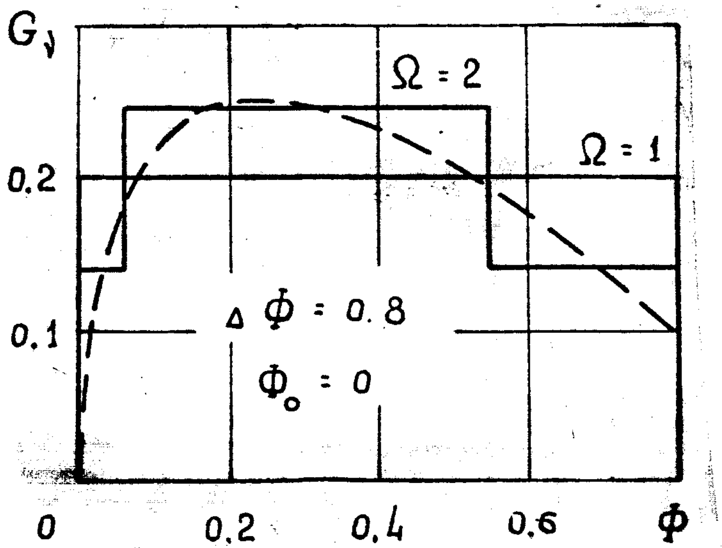

Figure 2

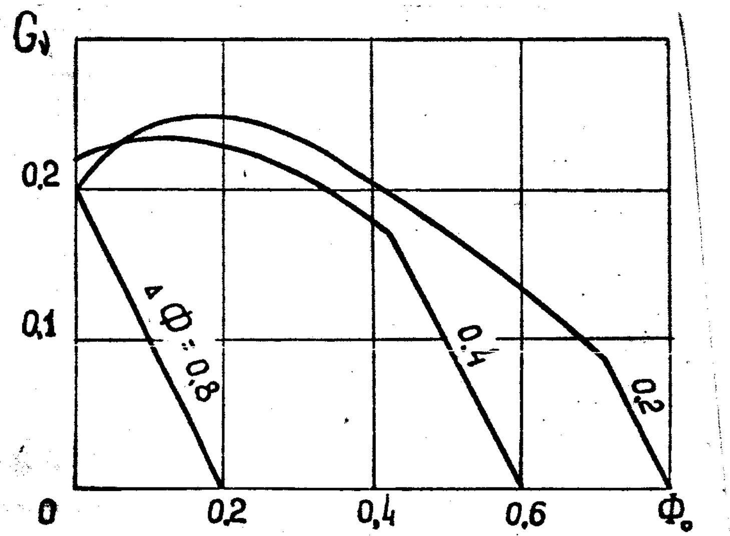

Figure 3

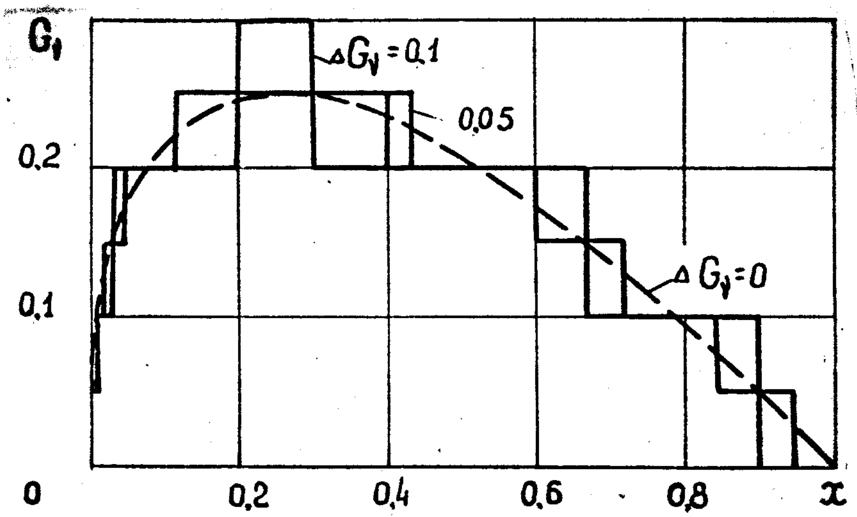

Figure 4

to zero (the dotted curves and $\Omega = 2$ curves, with the exception of $\Delta\Phi = 0.8$, are indistinguishable within the scale of the figure).

An instance of optimum distribution of the weight of propulsion system G_ν by maneuver by Φ), with $\Omega = 1.2, \infty$, is cited in figure 2. The $G_\nu^{(i)}(\Phi_0, \Delta\Phi)$ dependence is presented in figure 3 ($\Omega = 1$). The straight line segments correspond to the limitation of $G_\nu^{(i)} \leqslant 1 - \Phi_1$.

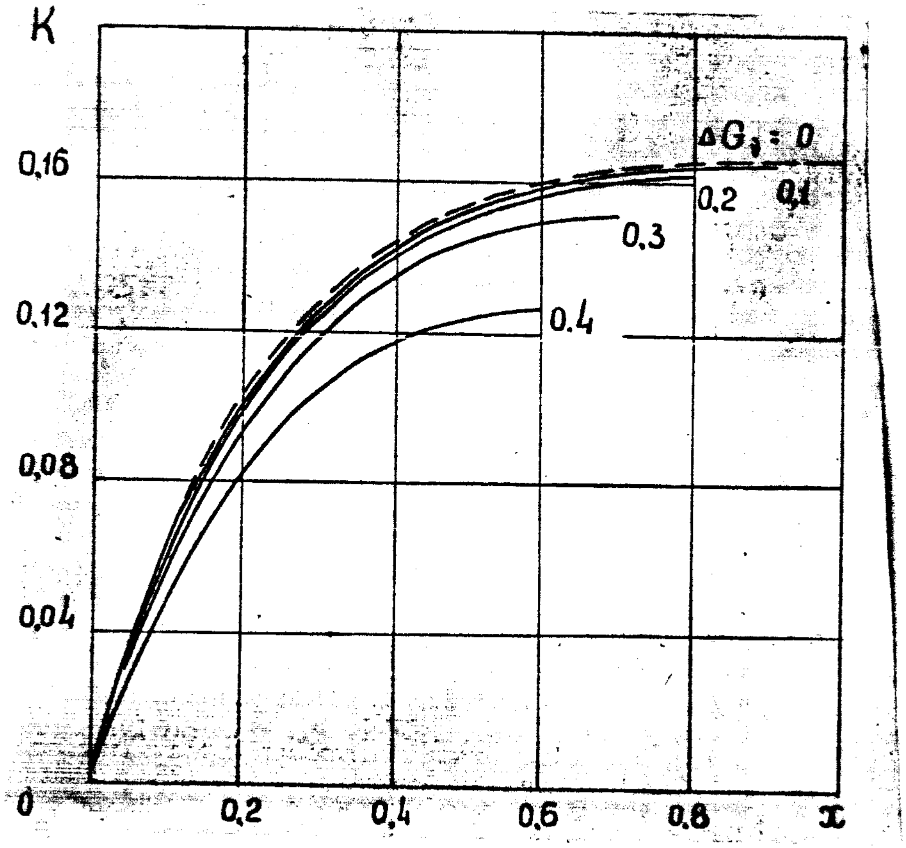

Figure 5

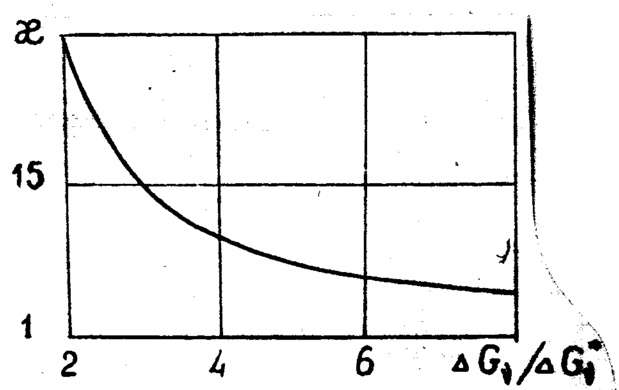

Figure 6

The second problem calls for the construction of a piecewise-continuous function $\sigma(\Phi)$ with integral levels, and the selection of the value of parameter ΔG_ν, ensuring a maximum of (5.14), on condition that the subintegral expression of (5.14) is nonnegative.

We will replace variable $x = \varkappa \Phi$ and introduce the following designation

$$K(x, \Delta G_\nu) = \int_0^x \sigma \Delta G_\nu \left(\frac{1}{\sigma \Delta G_\nu + x} - 1 \right) dx \qquad (5.15)$$

(5.14) will then be rewritten as follows

$$\langle G_\tau \rangle = \frac{1}{\varkappa(\Phi_1 - \Phi_0)} \left[K(\varkappa \Phi_1; \Delta G_\nu) - K(\varkappa \Phi_0, \Delta G_\nu) \right] \qquad (5.16)$$

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The (5.15) integral, just like the (5.13), is taken by segment between the σ_j level changes, and the moments of change are determined from the maximum subintegral expression of (5.15) by the σ_j integral values $/ \sigma = 1, 2, \ldots, \sigma_{max}; \sigma_{max} - E(1/4\Delta G_\nu)$ or $E(1/4\Delta G_\nu) + 1$, where E indicates the integral part of the number, see $\Delta G_\nu \sigma(\Phi)$ in figure 4). The K(x) dependence for various ΔG_ν values is shown in figure 5. It appears that, with $\Delta G_\nu < 0.1$, the curves practically coincide with the limiting dependence ($\Delta G_\nu \to 0$)

$$K(x, 0) = x \left(1 - \tfrac{4}{3}\sqrt{x} + \tfrac{1}{2} x \right) \qquad (5.17)$$

An approximate substitution of K (x, o) for K (x, ΔG_ν) in (5.16), (with $\Delta G_\nu < 0{,}1$) will produce

$$\langle G_\tau \rangle \approx 1 - \tfrac{4}{3}\sqrt{\varkappa}\, \frac{\Phi_1 + \sqrt{\Phi_1 \Phi_0} + \Phi_0}{\sqrt{\Phi_1} + \sqrt{\Phi_0}} + \tfrac{1}{2}\varkappa(\Phi_1 + \Phi_0) \qquad (5.18)$$

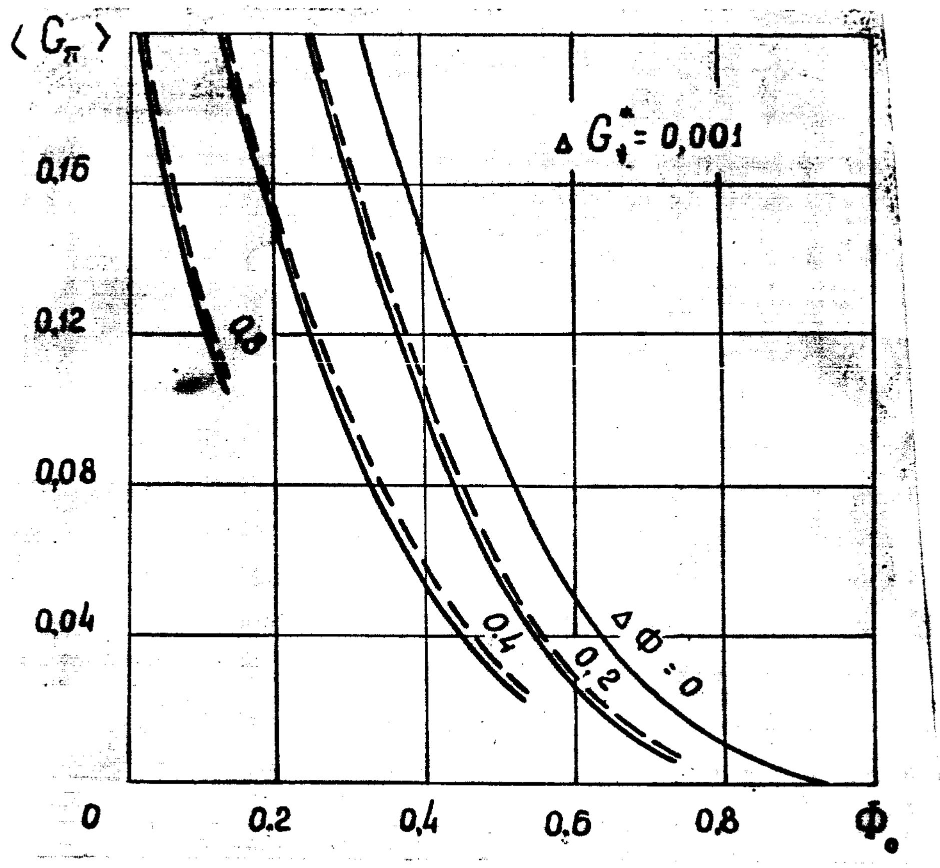

Figure 7

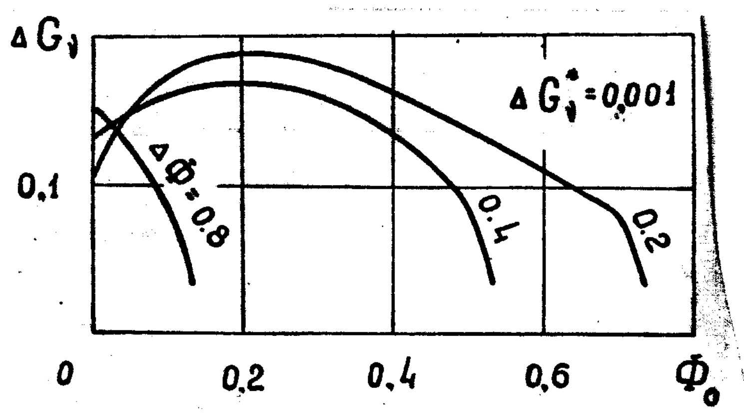

Figure 8

where $1 < \varkappa = \varkappa(\Delta G_y) \leq (1 - \Delta G_y)/\Phi_1$ (a limitation by $\varkappa$ from the top was made possible by the nonnegativity of subintegral expression (5.15)).

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The finding of ΔG_ν optimum value requires a given $\varkappa(\Delta G_\nu)$ dependence, for example

$$\varkappa = \Delta G_\nu / (\Delta G_\nu - \Delta G_\nu^*)$$

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where ΔG_ν^* is the specific gravity of the module with power converging to zero. The nature of this dependence coincides with those described in literature (see figure 6 and [1]).

The permissible change interval is determined from the $[\Phi_0, \Phi_1]$ range on the basis of the nonnegativity and payload for each maneuver $\varkappa$

$$\varkappa_1 < \varkappa < \varkappa_2 \quad \left(\varkappa_{1,2} = \frac{1}{2}\left[1 + \frac{1-\Delta G_\nu^*}{\Phi_1}\right] \mp \sqrt{\frac{1}{4}\left[1 + \frac{1-\Delta G_\nu^*}{\Phi_1}\right]^2 - \frac{1}{\Phi_1}}\right)$$

In this interval the $\langle G_\pi \rangle$ of (5.18) turns out to be a monotonically decreasing function of $\varkappa$. Hence the optimum $\varkappa$ and ΔG_ν values are equal to

$$\varkappa = \varkappa_1, \quad \Delta G_\nu = \Delta G_\nu^* \varkappa_1 / (\varkappa_1 - 1)$$

With $\Delta G_\nu = 0.1$, the problem can be solved numerically by the use of the known $K(x, \Delta G_\nu)$-dependence, presented in figure 5, and by formula (5.16). The results of the solution are shown in figures 7 and 8 ($\Delta G_\nu^* = 0.001$) which are similar to figures 1 and 2.

REFERENCE

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